



A Dictionary Based Image Coding using Orthogonal Matching Pursuit

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Abstract

This paper presents an efficient dictionary based image coding technique with static construction of over-complete dictionaries. For training, a static dictionary is adopted which is constructed by a combination of Gabor and Discrete Cosine Transform (DCT) basis function. The training samples are generated from the pixels value of 8X8 blocks of different standard images with varied pixel orientation. Further, the dictionary elements are encoded using orthogonal matching pursuit (OMP) algorithm. The popular transform based image coding is replaced by a dictionary based approach. It has been observed that the dictionary trained with the 8X8 blocks of standard images is capable of extracting the 8X8 blocks from other standard images which are not considered during the training. Coding results are presented to illustrate the performance of the proposed scheme in comparison to best existing coding solutions such as JPEG, JPEG 2000 etc. In general, it is observed that the proposed scheme has superior performance as compared to its competitive schemes.

Keywords: JPEG, JPEG 2000, Matching Pursuit, Orthogonal Matching Pursuit

1. Introduction

Image compression has become a basic essentiality in the field of digital image processing. It is the art of reducing the amount of data required to represent an image. Compression helps to save storage space and also reduce the transmission time required while enhancing the usage of available channel capacity. The original image can contain irrelevant and repeated information and image compression helps remove this redundancy of data and represents the information in a compact form.

In the recent years, orthogonal and bi-orthogonal complete dictionaries using DCT, wavelets etc. have been dominated in image and video coding standards. Recently sparse coding in redundant basis has attracted considerable interest in many areas of signal processing such as compression, de-noising, time frequency analysis, indexing etc. The conventional DCT complete dictionary suffers from blocking artifacts. Wavelet based compression is also unable to extract the directional information present in the image. So by analysis of all of the facts discussed above, in this paper we propose an efficient static dictionary based image coding algorithm to address the above problem occurring in DCT and wavelet. The dictionary can be trained by using the non-overlapping 8X8 blocks of image with different pixel orientation. To construct the dictionary, we designed an over complete static dictionary with combination of DCT and Gabor basis function. In this proposed algorithm the conventional DCT transform can be replaced by a set of trained dictionaries. To code the dictionary elements OMP algorithm is employed.



The rest of the paper is organized as follows. Section 2 gives the related works on sparse coding. In Section 3 the proposed static dictionary based image coding using OMP is introduced. Section 4 illustrates some selected results analysis & discussion and finally Section 5 gives the concluding remarks.

2. Related Work

This section discusses some of the notable research work carried out by several authors for improving image compression using sparse approximation technique. Mallat and Zhang [1] introduced the popular algorithm of this technique the Matching Pursuit (MP) algorithm as a greedy algorithm for time frequency analysis. The basic problem is to find a set of elements called as atoms from an over-complete dictionary, whose linear combination will efficiently approximate the given signal [2], [3]. This problem is a NP-hard problem. The matching pursuit algorithm is suitable for applications in areas like audio compression, image compression and video compression [4], [5], [6]. An image representation can be achieved using matching pursuit while retaining the important local properties like localization, scale, preferred orientation, amplitude and phase of discontinuity. For this purpose Mallat et al. constructed an extremely redundant dictionary of oriented Gabor functions and were able to represent the image in a compact form which could be used as input in sophisticated high level processing [7]. In [8] the author suggests that there has to be an optimization of the cost or bit rate associated with the selected vectors rather than the number of vectors that are selected. An improvement in performance is achieved when the selected vectors can be coded using the shortest code. It was applied to transform based coding and fractal coding. Liu et al. [9] proposed that the size of the dictionary should be determined such that it minimizes the number of bits required to represent the approximation of the input image. When we decompose a signal using matching pursuit we store the coefficients and the indices of the elements. Thus a dictionary of larger size will require more bits to represent the indices. In this paper [9] the authors provide with a formula that determines the optimal number of elements of the dictionary and the optimal quantization step that will reduce the number of bits required to store the matching pursuit representation to the minimum while considering the upper bound of error. In [10] the computational complexity of matching pursuit is significantly decreased by a proposed Fast M-fold Pursuit (FMFP) algorithm. It also achieves a good quality approximation.

3. MP and OMP Sparse Coding

This section describes some of the theoretical foundation of sparse coding. First, we have discussed MP in a nutshell followed by OMP which has been applied in our proposed approach.

3.1 Matching Pursuit

The matching pursuit algorithm solves the problem of decomposing a given signal over a redundant dictionary of atoms whose linear combination will produce an approximation of the image. The smallest possible dictionary will be the basis. However general dictionaries are redundant and thus give the freedom of choosing the signal's representation. There can be various presentation of the same image using different number of atoms selected from the dictionary. The chosen atoms should be such that they best represent the predominant features of the image without losing its psycho-visual property. Using this algorithm finding an exact solution requires an exhaustive combinatorial approach. So this algorithm uses a greedy approach to find the sub-optimal approximation. The details working principle of MP algorithm is discussed as follows.

Let a_i , $1 \leq i \leq N$ denote the i^{th} column of the dictionary matrix A . At the j^{th} iteration $j = 1, 2, \dots$ the algorithm finds,

$$atom_j = \arg \max_{a_i \in A} |\langle r_j - 1, a_i \rangle| \quad (1)$$

where, A denotes the dictionary of atoms, $r_j - 1$ denotes the approximation error or residual at the



$(j-1)^{th}$ iteration, and $\langle \cdot \rangle$ denotes the inner-product operation defined as $\langle u, v \rangle = u^T v$. The above step is known as the atom selection step. At the start of the iteration, the approximation error is equal to the given vector and hence $r_0 = b$. The weight or coefficient of the selected atom denoted as $atom_j$ is $\langle r_{j-1}, atom_j \rangle$ and let us denote it as c_j . The algorithm then updates the residual as

$$r_j = r_{j-1} - c_j atom_j \quad (2)$$

The above step is known as the residual update state. So, we will send the decoder a combination of (c, p) where c is the coefficient value and p is the position.

After some iteration the algorithm terminates if the norm of the residual falls below a desired approximation error bound, or if the number of distinct atoms in the approximation equals a desired limit.

3.2 Orthogonal Matching Pursuit

In standard matching pursuit, the orthogonalization of the residual with respect to each newly selected library atom can introduce components of previously selected atom into the residual. To avoid this problem, at each stage, instead of finding the new residual vector by orthogonalizing the old residual with respect to the newly selected library vector, we can orthogonalize x with respect to all the selected atom vectors. The new residual generated this way is orthogonal to all previously selected atoms and is therefore equal to the residual after finding the least squares solution using all the selected atom vectors. Compared to standard matching pursuit, this method usually results in a smaller number of selections. Like the MP, at the j^{th} iteration the algorithm first computes the atoms as

$$atom_j = \arg \max_{a_i \in A/A_{j-1}} |\langle r_j - 1, a_i \rangle| \quad (3)$$

where, A denotes the dictionary of atoms, $r_j - 1$ denotes the approximation error or residual at the $(j-1)^{th}$ iteration, and $'\setminus'$ denotes the set difference operator. This is the atom selection step. The approximation at the j^{th} iteration is given as the orthogonal projection of the original signal vector onto the subspace spanned by the selected atoms. Then the approximation at the j^{th} iteration is given as

$$bj = A_j (A_j^T A_j)^{-1} A_j^T b = A_j c_j \quad (4)$$

Where, c_j denotes the coefficient vector at the j^{th} iteration, which is nothing but the solution vector obtained using the pseudo-inverse of A_j . Instead of computing c_j as above, it is less complex to derive it through a QR factorization. In the second step, the algorithm updates the residual as

$$r_j = b - A_j c_j \quad (5)$$

The algorithm terminates if the norm of the residual falls below a desired approximation error bound, or if the number of atoms in the approximation equals a desired limit. Otherwise, it proceeds to the next iteration. The nonzero components of the sparse solution vector resulting from the OMP are equal to the components of the coefficient vector at the last iteration. The OMP is seemingly very simple but is actually very powerful.

4. Proposed Dictionary Based Image Coding

In sparse approximation technique, the problem is to represent the input image using minimum number of atoms from an over-complete dictionary. The DCT based dictionary will select the horizontal and vertical components efficiently. However the directional components of an image may not be properly approximated from a dictionary constructed using only a DCT-based dictionary. An image usually contains very different types of features, which have been successfully modelled by the very redundant family of 2D Gabor oriented wavelets, describing the local properties of the image: localization, scale, preferred orientation, amplitude and phase of the discontinuity. However, this model generates representations of very large size. Instead of decomposing a given image over this whole set of Gabor functions. The Gabor functions take into account the directional features of the image. Image decompositions in families of Gabor functions characterize the local

scale, orientation and phase of the image variations. Gabor functions are constructed from a window, modulated by sinusoidal waves of fixed frequency w_0 that propagate along different direction with two different phases 0 and 90 degrees. Each of these modulated windows can be interpreted as wavelets having different orientation. Thus a dictionary designed from the combination of DCT and Gabor transform will consider the vertical, horizontal as well as the directional components efficiently in the given image. This will represent the image in a compact way while maintaining the information content present in it. Figure 1 and Figure 2 show the generic model of the encoder and decoder of the proposed approach. The overall steps of the proposed approach are discussed in a sequel.

1. Select the pixels of non-overlapping block of size 8X8 of different standard images with varies pixel orientation.
2. Arrange the 8X8 blocks of an image in a column format as the elements of the proposed static based dictionary. To encode the dictionary element employed OMP algorithm.
3. Update the dictionary element using the proposed dictionary. Continue updating process till no further change of element in the dictionary.
4. Select the coefficients value and position (c,p) of the dictionary element and transmit those values to the decoder. Employ uniform quantization and entropy coding to the coefficients. Encode the position using fixed length code.
5. After the selection of the dictionary elements, the coefficients are quantized and entropy coded .
6. Apply inverse quantization to reconstruct the 8X8 blocks of an image.

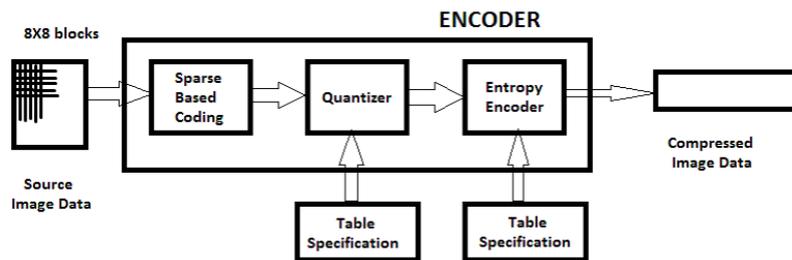


Figure 1 (Block diagram of the encoder of the proposed model)

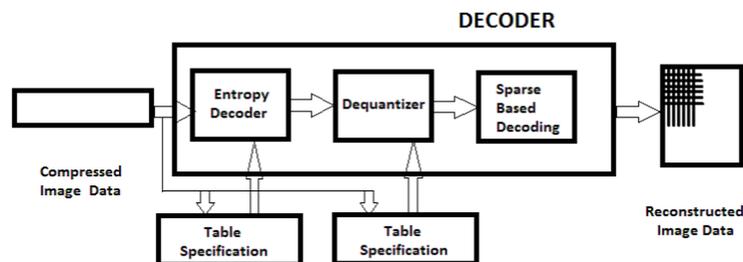


Figure 2 (Block diagram of the decoder of the proposed model)

5. Results & Discussion

In order to evaluate the efficiency of the proposed scheme different standard images such as *Football*, *Peppers*, *Gantrycrane*, *Green* etc. are considered in the experiment. In the experiment four schemes i.e. JPEG, JPEG 2000, DCT dictionary based image coding scheme, HARR dictionary based image coding scheme along with the proposed scheme were implemented using Matlab. The images used during our experiments are of 256X256 pixels with 8bit gray level quantization. In the experiment, the over complete proposed dictionaries are trained with several standard images such as *Football*, *Peppers*, *Gantrycrane*, and *Green*. The size of the dictionary of the proposed scheme is 64X128 where 64 represents the column that is the total number of pixels in a block and the rows represents the number of dictionary elements. Figure 3 shows the bit rate vs PSNR plot of the proposed scheme as compared to JPEG, JPEG 2000, DCT-based dictionary image coding scheme and HARR-based dictionary image coding scheme. The proposed algorithm yields an average peak signal to noise ratio (PSNR) gain of 3.5dB, 0.5dB, 0.4dB and 0.7dB as compared to JPEG, JPEG 2000, DCT-based dictionary image coding scheme and HARR-based dictionary image coding scheme for the *Peppers* image. Figure 4- Figure 8 show the reconstructed images of the JPEG, JPEG 2000, DCT-based dictionary image coding scheme, HARR-based dictionary image coding scheme and the proposed scheme. Table 1 shows the comparison of PSNR in dB of different standard images. The proposed scheme yields 36.1 dB PSNR for the image *Peppers*. The gain of PSNR in dB of the proposed model for the image *Peppers* over JPEG, JPEG 2000, DCT-based dictionary image coding scheme, HARR-based dictionary image coding scheme are 3.972 dB, 3.552 dB, 3.3882 dB, and 2.37 dB respectively considered for 600 bit rate.

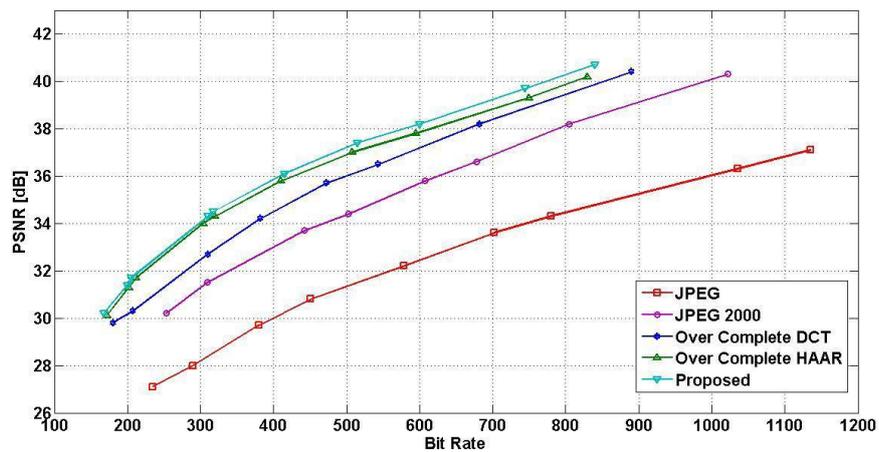


Figure 3: Bit rate vs PSNR (in dB) of *Pepper* image



Figure 4: JPEG Scheme



Figure 5: JPEG-2000 Scheme



Figure 6: DCT Dictionary Scheme



Figure 7: HAAR Dictionary Scheme



Figure 8: Proposed Dictionary Scheme

Table 1: PSNR in (dB) Obtained for Different Standard Images (for 600 kbps)

Image Used	JPEG	JPEG-2000	DCT Dictionary	HAAR Dictionary	Proposed Dictionary
Football	28.63	28.973	29.2831	29.45	30.45
Peppers	32.128	32.5478	32.7118	33.73	36.1
Gantrycrane	28.34	28.543	29.0091	29.9863	31.2
Green	24.328	24.774	25.1333	25.5541	26.1



6. Conclusion

In this paper, we propose an efficient over-complete static dictionary for image compression. The proposed algorithm trains the static over-complete dictionary with the combination of DCT and Gabor basis. The dictionary elements are trained using OMP algorithm. Experimental results show the superiority of the proposed scheme as compared to its counter parts. Further, it has been observed that the proposed dictionary based image coding shows significant compression efficiency and improved PSNR gain as compared to the best existing schemes available in the literature

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